HONORS Extended HW Week 20
Linear and Angular Speed

Name______________________________
Date__________________________ Per___

Velocity:

- Velocity is the speed of an object in a given direction. For example, a car might have a velocity (think speed) of 25 miles per hour. The formula for velocity is: $v = \frac{d}{t}$ where d is distance and t is time.

1) While on vacation, Javier traveled a total distance of 440 miles. His trip took 8 hours. What was his average speed?

Arc Length:

- Read page 492 "The Length of a Circular Arc" and "Example 8 Finding the Length of a Circular Arc"

2) Why is the formula for finding the length of an arc $s = r\theta$? Draw a picture, if needed.

3) Solve "Check Point 8" on page 493.

Linear and Angular Speed:

- Read page 493

4) What is linear speed? What is the formula for calculating linear speed? Is linear speed the same as velocity mentioned above? Draw a picture, if needed.

5) What is angular speed? What is the formula for calculating angular speed?

6) Why is the formula for linear speed in terms of angular speed $v = r\omega$?

7) Why is one revolution is equivalent to 2π?

8) Solve "Check Point 9" on page 494.

9) Create your own word problem involving linear speed and solve it below. Draw a picture, if needed.

5 Find the Linear Speed of an Object Traveling in Circular Motion

The average speed of an object equals the distance traveled divided by the elapsed time. For motion along a circle, we distinguish between **linear speed** and **angular speed.**

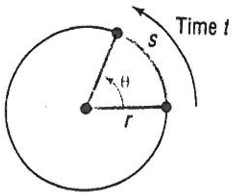
DEFINITION

Suppose that an object moves around a circle of radius r at a constant speed. If s is the distance traveled in time t around this circle, then the **linear speed** v of the object is defined as

$$v = \frac{s}{t} \tag{9}$$

Figure 16

$v = \frac{s}{t}$



As this object travels around the circle, suppose that θ (measured in radians) is the central angle swept out in time t. See Figure 16.

DEFINITION

The **angular speed** ω (the Greek letter omega) of this object is the angle θ (measured in radians) swept out, divided by the elapsed time t; that is,

$$\omega = \frac{\theta}{t} \tag{10}$$

Angular speed is the way the turning rate of an engine is described. For example, an engine idling at 900 rpm (revolutions per minute) is one that rotates at an angular speed of

$$900\frac{\text{revolutions}}{\text{minute}} = 900\frac{\cancel{\text{revolutions}}}{\text{minute}}\cdot 2\pi\frac{\text{radians}}{\cancel{\text{revolution}}} = 1800\pi\frac{\text{radians}}{\text{minute}}$$

There is an important relationship between linear speed and angular speed:

$$\text{linear speed} = v \underset{(9)}{=} \frac{s}{t} \underset{s = r\theta}{=} \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) \underset{(10)}{=} r\cdot\omega$$

$$v = r\omega \tag{11}$$

where ω is measured in radians per unit time.

When using equation (11), remember that $v = \frac{s}{t}$ (the linear speed) has the dimensions of length per unit of time (such as feet per second or miles per hour), r (the radius of the circular motion) has the same length dimension as s, and ω (the angular speed) has the dimensions of radians per unit of time. If the angular speed is given in terms of *revolutions* per unit of time (as is often the case), be sure to convert it to *radians* per unit of time using the fact that 1 revolution $= 2\pi$ radians before attempting to use equation (11).

EXAMPLE 8 **Finding Linear Speed**

A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.

Solution Look at Figure 17. The rock is moving around a circle of radius $r = 2$ feet. The angular speed ω of the rock is

$$\omega = 180\frac{\text{revolutions}}{\text{minute}} = 180\frac{\cancel{\text{revolutions}}}{\text{minute}}\cdot 2\pi\frac{\text{radians}}{\cancel{\text{revolution}}} = 360\pi\frac{\text{radians}}{\text{minute}}$$

Solution

a. For a 750° angle, subtract two multiples of 360°, or 720°, to find a positi
coterminal angle less than 360°.

$$750° - 360° \cdot 2 = 750° - 720° = 30°$$

A 30° angle is coterminal with a 750° angle.

b. For a $\frac{22\pi}{3}$, or $7\frac{1}{3}\pi$, angle, subtract three multiples of 2π, or 6π, to find
positive coterminal angle less than 2π.

$$\frac{22\pi}{3} - 2\pi \cdot 3 = \frac{22\pi}{3} - 6\pi = \frac{22\pi}{3} - \frac{18\pi}{3} = \frac{4\pi}{3}$$

A $\frac{4\pi}{3}$ angle is coterminal with a $\frac{22\pi}{3}$ angle.

c. For a $-\frac{17\pi}{6}$, or $-2\frac{5}{6}\pi$ angle, add two multiples of 2π, or 4π, to find a posi
coterminal angle less than 2π.

$$-\frac{17\pi}{6} + 2\pi \cdot 2 = -\frac{17\pi}{6} + 4\pi = -\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$$

A $\frac{7\pi}{6}$ angle is coterminal with a $-\frac{17\pi}{6}$ angle.

Discovery

Make a sketch for each part of Example 7 illustrating that the coterminal angle we found and the given angle have the same initial and terminal sides.

Check Point 7 Find a positive angle less than 360° or 2π that is cotern
with each of the following:

a. an 855° angle **b.** a $\frac{17\pi}{3}$ angle **c.** a $-\frac{25\pi}{6}$ angle.

7 Find the length of a circular arc.

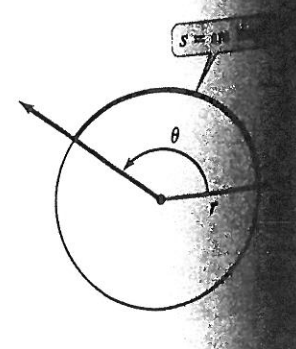
The Length of a Circular Arc

We can use the radian measure formula, $\theta = \frac{s}{r}$, to find the length of the arc of
How do we do this? Remember that s represents the length of the arc interc
the central angle θ. Thus, by solving the formula for s, we have an equation
length.

The Length of a Circular Arc

Let r be the radius of a circle and θ the nonnegative radian measure of a central angle of the circle. The length of the arc intercepted by the central angle is

$$s = r\theta.$$



EXAMPLE 8 **Finding the Length of a Circular Arc**

A circle has a radius of 10 inches. Find the length of the arc intercepted by
angle of 120°.

Solution The formula $s = r\theta$ can be used only when θ is expressed in radians. Thus, we begin by converting 120° to radians. Multiply by $\frac{\pi \text{ radians}}{180°}$.

$$120° = \cancel{120°} \cdot \frac{\pi \text{ radians}}{\cancel{180°}} = \frac{120\pi}{180} \text{ radians} = \frac{2\pi}{3} \text{ radians}$$

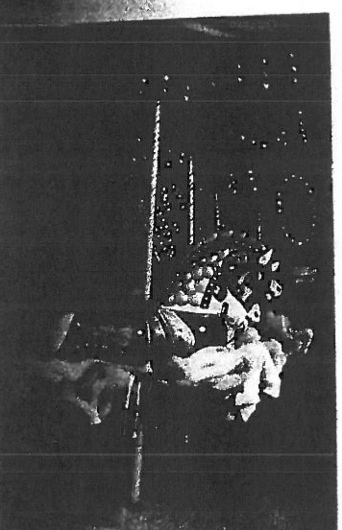
... to describe the length ... arc is the same unit that ... circle's radius.

Now we can use the formula $s = r\theta$ to find the length of the arc. The circle's radius is 10 inches: $r = 10$ inches. The measure of the central angle, in radians, is $\frac{2\pi}{3}$: $\theta = \frac{2\pi}{3}$. The length of the arc intercepted by this central angle is

$$s = r\theta = (10 \text{ inches})\left(\frac{2\pi}{3}\right) = \frac{20\pi}{3} \text{ inches} \approx 20.94 \text{ inches.}$$

Check Point 8 A circle has a radius of 6 inches. Find the length of the arc intercepted by a central angle of 45°. Express arc length in terms of π. Then round your answer to two decimal places.

... ear and angular speed ... scribe motion on a ... path.



Linear and Angular Speed

A carousel contains four circular rows of animals. As the carousel revolves, the animals in the outer row travel a greater distance per unit of time than those in the inner rows. These animals have a greater *linear speed* than those in the inner rows. By contrast, all animals, regardless of the row, complete the same number of revolutions per unit of time. All animals in the four circular rows travel at the same *angular speed.*

Using v for linear speed and ω (omega) for angular speed, we define these two kinds of speeds along a circular path as follows:

Definitions of Linear and Angular Speed

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its **linear speed** is

$$v = \frac{s}{t},$$

where s is the arc length given by $s = r\theta$, and its **angular speed** is

$$\omega = \frac{\theta}{t}.$$

The hard drive in a computer rotates at 3600 revolutions per minute. This angular speed, expressed in revolutions per minute, can also be expressed in revolutions per second, radians per minute, and radians per second. Using 2π radians $= 1$ revolution, we express the angular speed of a hard drive in radians per minute as follows:

$$\begin{aligned} &3600 \text{ revolutions per minute} \\ &= \frac{3600 \cancel{\text{ revolutions}}}{1 \text{ minute}} \cdot \frac{2\pi \text{ radians}}{1 \cancel{\text{ revolution}}} = \frac{7200\pi \text{ radians}}{1 \text{ minute}} \\ &= 7200\pi \text{ radians per minute.} \end{aligned}$$

We can establish a relationship between the two kinds of speed by dividing both sides of the arc length formula, $s = r\theta$, by t:

$$\underbrace{\frac{s}{t}}_{\text{This expression defines linear speed.}} = \frac{r\theta}{t} = r\underbrace{\frac{\theta}{t}}_{\text{This expression defines angular speed.}}.$$

Thus, linear speed is the product of the radius and the angular speed.

Linear Speed in Terms of Angular Speed

The linear speed, v, of a point a distance r from the center of rotation is given by

$$v = r\omega,$$

where ω is the angular speed in radians per unit of time.

EXAMPLE 9 **Finding Linear Speed**

A wind machine used to generate electricity has blades that are 10 feet in length (see **Figure 5.18**). The propeller is rotating at four revolutions per second. Find the linear speed, in feet per second, of the tips of the blades.

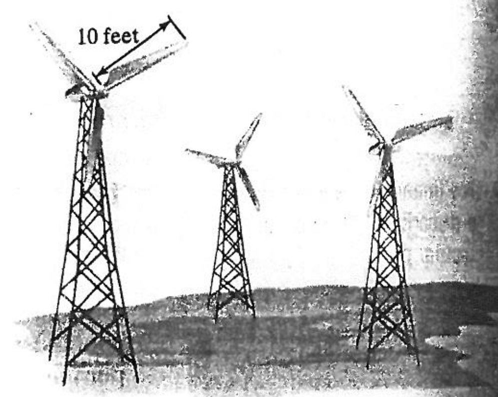


Figure 5.18

Solution We are given ω, the angular speed.

$\omega = 4$ revolutions per second

We use the formula $v = r\omega$ to find v, the linear speed. Before applying the formula, we must express ω in radians per second.

$$\omega = \frac{4 \cancel{\text{revolutions}}}{1 \text{ second}} \cdot \frac{2\pi \text{ radians}}{1 \cancel{\text{revolution}}} = \frac{8\pi \text{ radians}}{1 \text{ second}} \quad \text{or} \quad \frac{8\pi}{1 \text{ second}}$$

The angular speed of the propeller is 8π radians per second. The linear speed is

$$v = r\omega = 10 \text{ feet} \cdot \frac{8\pi}{1 \text{ second}} = \frac{80\pi \text{ feet}}{\text{second}}.$$

The linear speed of the tips of the blades is 80π feet per second, which is approximately 251 feet per second.

Check Point 9 Long before iPods that hold thousands of songs and play them with superb audio quality, individual songs were delivered on 75-rpm and 45-rpm circular records. A 45-rpm record has an angular speed of 45 revolutions per minute. Find the linear speed, in inches per minute, at the point where the needle is 1.5 inches from the record's center.

Exercise Set 5.1

Practice Exercises

In Exercises 1–6, the measure of an angle is given. Classify the angle as acute, right, obtuse, or straight.

1. $135°$ **2.** $177°$

3. $83.135°$ **4.** $87.177°$

5. π **6.** $\frac{\pi}{2}$

In Exercises 7–12, find the radian measure of the central angle of a

Radius, r	Arc Length, s
7. 10 inches	40 inches
8. 5 feet	30 feet
9. 6 yards	8 yards
10. 8 yards	18 yards
11. 1 meter	400 centimeters
12. 1 meter	600 centimeters